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Operational Availability of a Power Distribution System with Environmental Effects under Preemptive Repeat Repair Policy

Sapna Jain and Ajay Kumar Singhal

ABSTRACT : This paper deals with operational availability of a three state power distribution system consisting of two subsystems with environmental effects under preemptive repeat repair policy. A mathematical model for a power distribution system under service restrictions has been developed for exponential failures and general repairs. Supplementary variable technique has been employed to obtain various state probabilities and then the operational availability is obtained by the inversion process. The study state behavior, reliability, and MTTF have also been analyzed by some graphical illustrations to explain the practical utility of the model.

Keywords : MTTF, operational availability, Power distribution system, preemptive repeat repair policy, supplementary variable technique.

M, N	:	No of units in subsystem A and B respectively
$\lambda' / \lambda_i / \upsilon$	:	Constant failure rates of any unit of A/ith unit of B/ Environmental failure
$\eta_i(x) / S_i(x)$	:	Transition repair rate of the subsystem B/pdf of repair rate
$\phi_j(y) / S_j(y)$	:	Transition repair rate of the subsystem A/pdf of repair rate
δ	:	Transition repair rate of Environmental failure
$P_{M,N}(t)$	:	The probability that at time t the system is operating in the state of normal efficiency
$P_{M-r,N}(t)$	:	The probability that at time t the system is in operable state where in r components of subsystem A have already failed.
$P_{M-j,N}(y,t)\Delta$	:	The probability that at time t the system is operating in reduced efficiency due to the failure of j components of subsystem A. The elapsed repair time lies in the interval $(y, y+\Delta)$
$P_{M,F_i}(x,t)\Delta$	:	The probability that at time t the system is in the failed state due to the failure of ith component of subsystem B and all M components of A are in operable state and Elapsed repair time lies in the interval $(x, x+\Delta)$
$P_{M-r,F_i}(x,t)\Delta$	:	The probability that at time t the system is in the failed state due to the failure of ith component of subsystem B and $(M-r)$ components of subsystem A are in operable state and elapsed repair time for subsystem B lies in the interval $(x, x+\Delta)$ conditioned that no repair is carried out for subsystem A.
$Q_{M-j,F_i}(x,t)\Delta$	:	The probability that at time t, the system is in failed state due to failure of ith component of subsystem B and j components of subsystem A, the elapsed repair time for subsystem B lies in the interval $(x, x+\Delta)$ at the instant when subsystem B preempted in the repair facility and subsystem A which had been in service is awaiting repair.
$P_F(t)\Delta$	:	The probability that at time t the system is in the failed state due to the environmental failure.
λ	:	$\sum_{i=1}^{N} \lambda_i$
$S_i(x)$	:	$i(x)\exp\left[-\int_0^x i(x)dx\right]$ by Devis formula ,where $i = \eta, \phi$

- *Sapna Jain- Assistant Professor, Department of mathematics, JSS academy of technical education, Noida, India.*
- *Ajay Kumar Singhal -Larsen &Toubro Ltd., New Delhi, India.*

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INTRODUCTION

We have analyzed the operational availability of a power distribution system under the environmental effects and divided the complete power distribution system [23, 28] into two subsystems A and B as shown in fig.1 in which A consists of M identical power plants in parallel redundancy such that M-1 components are redundant components. B consist of N components as transmission substations, High voltage transmission lines, power substation,transformer, power plants and transformer drum in series such that a failure in any component brings about the complete breakdown of the system whereas a failure of $j(\geq 2, < M)$ components of subsystem A causes the system to work in reduced efficiency state.

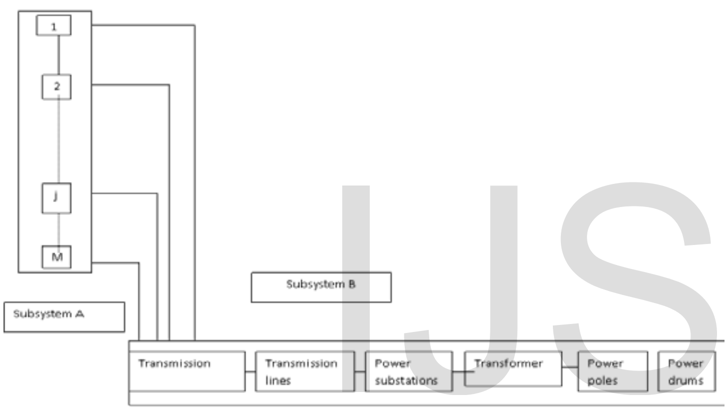

Fig. 1 **Block Diagram of Power distribution system**

As well M-power plants is j-out of-M: G system (More than $M - j + 1$ power plants must fail for A to fail) also the distribution system can fail due to environmental failure (like lightening) from the normal efficiency state the life times of active units depend on each other in having simultaneous failure of all the operating units and repair times are distributed quite generally. A failed unit is repaired at a single service channel.

The author has initiated the study of the operational availability of a power distribution system with environmental effects under preemptive repeat repair policy. In this policy, the component of subsystem A which has been removed from repair policy, due to the priority given to the repair of a component of subsystem B, when taken up again for repair, the repair already carried out on the component of subsystem A goes waste. In other words the repairs of the components of subsystem A, when restart is considered as a fresh failure All the assumptions are same as in the state transition diagram of the system with repair and failure rates under preemptive repeat repair policy is shown in fig.2.

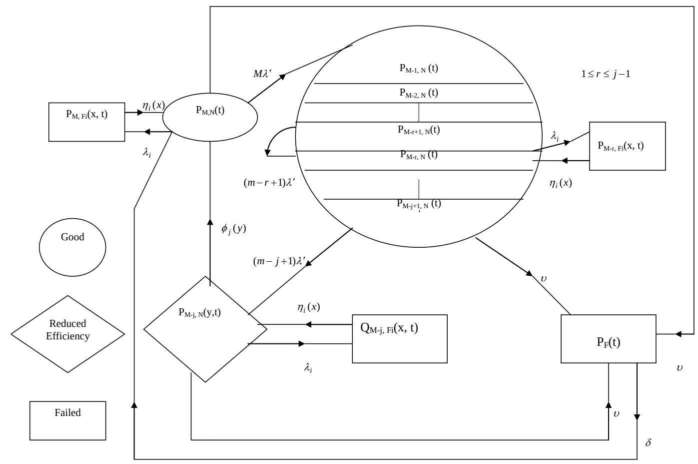

Fig.2 STATE TRANSITION DIAGRAM

II.FORMULATION OF MATHEMATICAL MODEL

The model under consideration is exhibited in fig. 1. The flow of states of the system under consideration has been depicted in fig. 2. The probabilities given above are mutually exclusive and provide the complete Markovion characteristics of the process. Therefore, using continuity arguments and elementary probability considerations one obtains the following forward difference differential equations governing the stochastic behavior of the complex system which is discrete in space and continuous in time:

$$\left[\frac{d}{dt}+\lambda+M\lambda'+\upsilon\right]P_{M,N}(t)=\int_0^\infty \eta_i(x)P_{M,F_i}(x,t)dx+\int_0^\infty \phi_j(y)P_{M-j,N}(y,t)dy+\delta P_{F(t)}$$

………. (1)

$$\left[\frac{d}{dt}+\lambda+(M-r)\lambda'+\upsilon\right]P_{M-r,N}(t)=(M-r+1)\lambda' P_{M-r+1,N}(t)+\sum_{i=1}^{N}\int_0^\infty P_{M-r,Fi}(x,t)\eta_i(x)dx$$

………. (2)

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\lambda+\phi_j(y)+\upsilon\right]P_{M-j,N}(y,t)=0$$

……… (3)

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\eta_i(x)\right]P_{M,Fi}(x,t)=0$$

………. (4)

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\eta_i(x)\right]P_{M-r,Fi}(x,t)=0$$

………. (5)

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\eta_i(x)\right]Q_{M-j,Fi}(x,t)=0$$

………. (6)

$$\left[\frac{d}{dt}+\delta\right]P_F(t)=\upsilon\left[P_{M-r,N}(t)+P_{M,N}(t)+\int_0^\infty P_{M-j,N}(y,t)dy\right] \quad \text{.......... (7)}$$

Boundary Conditions

$$P_{M,Fi}(0,t)=\lambda_i P_{M,N}(t) \quad \text{.......... (8)}$$

$$P_{M-j,N}(0,t)=(M-j+1)\lambda' P_{M-j+1,N}(t)+\sum_{i=1}^{N}\int_0^\infty Q_{M-j,Fi}(x,t)\eta_i(x)dx \quad \text{.......... (9)}$$

$$P_{M-r,Fi}(0,t)=\lambda_i P_{M-r,N}(t) \quad \text{.......... (10)}$$

$$Q_{M-j,Fi}(0,t)=\lambda_i\int_0^\infty P_{M-j,N}(y,t)dy \quad \text{.......... (11)}$$

Initial Condition

$P_k(0)=1$ as $K=M,N$ otherwise 0 .... (12)

These difference differential equations are solved by Laplace Transform Technique [ref 9] by using initial and boundary conditions obtained by state transition diagram [fig(2),ref 11] and then the Laplace Transform of operational availability [ref 5] is obtained. We can write it as

2.3.Evaluation of Laplace transform of up and down state probabilities

$$\overline{P_{up}(s)}=\frac{1}{D(s)}\left[1+T[s,r]+T[s,j]/(s+\lambda+\upsilon+\phi_j-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i})\right] \quad \text{............ (13)}$$

$$\overline{P_{down}(s)}=\frac{1}{s}-\overline{P}_{up}(s) \quad \text{............. (14)}$$

where,

$$T[s,r]=(\lambda')^r\prod_{p=1}^{r-1}\frac{M-P}{[s+\lambda+\upsilon-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}+(M-P-1)\lambda']}$$

$$E[s,j]=(M-j+1)(\lambda')^j\prod_{p=1}^{j-1}\frac{M-p+1}{[s+\lambda+\upsilon+(M-p)\lambda'-\Sigma\lambda_i\overline{S}_i(s)]}$$

$$D(s)=s+\lambda+M\lambda'+\upsilon-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}-\frac{\phi_j T[s,j]}{[s+\lambda+\upsilon+\phi_j-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}]}$$

$$-\frac{\upsilon\delta}{s+\delta}\left[1+T[s,r]+T(s,j)/(+\lambda+\upsilon+\phi_j-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i})\right]$$

Ergodic behavior

Using abel's lemma in Laplace transform, viz;

$\lim_{s\to 0} s\overline{f(s)}=\lim_{t\to\infty} f(t)=f$ (say)

Provided the limit on the right hand side exist, the time independent follows:

P_{up} = Point wise availability at $t\to\infty$

$$P_{up}=avss=\frac{1}{B}\left[\sum_{r=1}^{j-1}(\lambda')^r\prod_{p=0}^{r-1}\frac{M-p}{(\upsilon+(M-p-1)\lambda'}\sum_{i=1}^{r-1}\frac{1+\Sigma\lambda_i M_i}{[\upsilon+(M-p-1)\lambda']}+X\right] \quad \text{........... (15)}$$

$$P_{down}=1-P_{up} \quad \text{........... (16)}$$

where,

$$B=B'(0)=1-\Sigma\lambda_i M_i-X+\frac{\upsilon}{\delta}\left[(\lambda')^r\prod_{p=0}^{r-1}\frac{M-p}{(\upsilon+(M-p-1)\lambda')}+1\right]$$

$$+(M-j+1)(\lambda')^j\prod_{p=1}^{j-1}\frac{M-p-1}{[\upsilon+(M-p)\lambda']}\frac{\overline{D}_j(\lambda+\upsilon)}{[1-(\lambda+\upsilon)^{-1}[1-\overline{S}_j(\lambda+\upsilon)]\lambda]}]$$

$$-\upsilon\left[(\lambda')^r\prod_{p=0}^{r-1}\frac{M-p}{(\upsilon+(M-p-1)\lambda')}\sum_{i=1}^{r-1}\frac{1+\Sigma\lambda_i M_i}{(\upsilon+(M-p-1)\lambda'}-X\right]$$

$$X=\frac{(M-j+1)(\lambda')^j}{[1-(\lambda+\upsilon)^{-1}(1-\overline{S}_j(\lambda+\upsilon))\lambda]}\prod_{p=1}^{j-1}\frac{M-p+1}{[\upsilon+(M-p)\lambda']}\left\{\overline{S}'_j(\lambda+\upsilon)+\overline{S}_j(\lambda+\upsilon)\sum_{i=1}^{j-1}\frac{1+\Sigma\lambda_i M_i}{(\upsilon+(M-p)\lambda')}\right.$$

$$\left.-\frac{\overline{S}_j(\lambda+\upsilon)}{[1-(\lambda+\upsilon)^{-1}[1-\overline{S}_j(\lambda+\upsilon)]\lambda]}\left[(\lambda+\upsilon)^{-1}[1-\overline{S}_j(\lambda+\upsilon)\right]\Sigma\lambda_i M_i-\frac{1}{(\lambda+\upsilon)^2}(1-\overline{S}_j(\lambda+\upsilon))\lambda]\right\} \quad (13)$$

When repair follow exponential time distribution i.e. setting

$\overline{S}_i(s)=\frac{\eta_i}{s+\eta_i}$ and $\overline{S}_j(s)=\frac{\phi_j}{s+\phi_j}$ in the relation (15)

through (16), one gets:

$$\overline{P_{up}(s)}=\frac{1}{D(s)}\left[1+T[s,r]+T[s,j]/(s+\lambda+\upsilon+\phi_j-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i})\right] \quad \text{............... (17)}$$

$$\overline{P_{down}(s)}=\frac{1}{s}-\overline{P_{up}}(s) \quad \text{................. (18)}$$

$$T[s,r]=(\lambda')^r\prod_{p=1}^{r-1}\frac{M-P}{[s+\lambda+\upsilon-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}+(M-P-1)\lambda']}$$

where,

$$T[s,j]=(M-j+1)(\lambda')^j\prod_{p=0}^{j-1}\frac{M-P+1}{[s+\lambda+\upsilon+(M-p)\lambda'-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}]}$$

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$$D(s) = s+\lambda+M\lambda'+\upsilon-\sum_i \frac{\lambda_i\eta_i}{s+\eta_i}-\frac{\phi_j T[s,j]}{[s+\lambda+\upsilon+\phi_j-\sum_i \frac{\lambda_i\eta_i}{s+\eta_i}]}$$

and

$$-\frac{\upsilon\delta}{s+\delta}\left[1+T[s,r]+T(s,j)/(+\lambda+\upsilon+\phi_j-\sum_i \frac{\lambda_i\eta_i}{s+\eta_i})\right]$$

Reliability analysis

The Laplace transform of the reliability function when all repairs are zero of the system is given by

$$\overline{R(s)} = \frac{2}{s+\lambda+M\lambda'+\upsilon}-\frac{1}{s+\lambda+\upsilon} \qquad \text{......... (19)}$$

MTTF analysis

Mean time to failure is given by

$$= \frac{2}{\lambda+M\lambda'+\upsilon}-\frac{1}{\lambda+\upsilon} \qquad \text{......... (20)}$$

Numerical Computations

Inserting $M = 4, j = 2, \lambda = 0.3, \lambda' = .02, \upsilon = .01$ and $\eta = \phi = \delta = 1$, when all repair rates are equal then we get the following relations:-

Reliability function

$$R(t) = 2e^{-(.31+.00123M)t} - e^{-.31t} \qquad \text{.......... (21)}$$

Operational availability

$$P_{up}(t) = .668976546 + .363112193e^{-.255324t} - .058924046e^{-1.053051t} - .0$$
$$-.071789074e^{-.5641725t}\cos(.916573054t) - .348156148e^{-.5641725t}\sin(.916573054t) \qquad \text{......... (22)}$$

$$P_{down}(t) = 1 - P_{up}(t) \qquad \text{......... (23)}$$

Mean time to failure

$$\text{MTTF} = \frac{.31 - M\lambda'}{(.0961 + .31M\lambda')} \qquad \text{......... (24)}$$

Graphical Representation

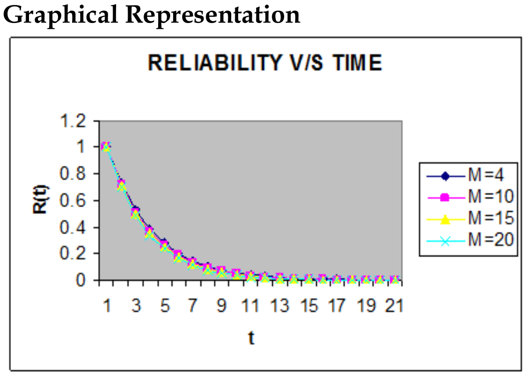

Fig (3): reliability vs. time

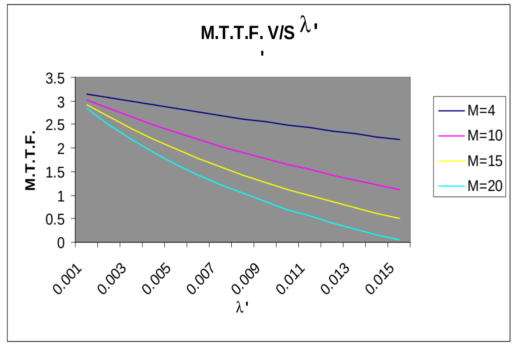

Fig (4): MTTF vs. failure

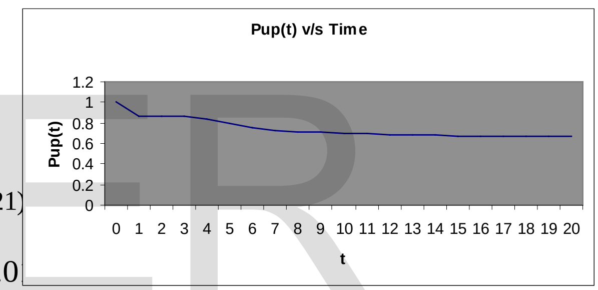

Fig (5): operational availability Vs. time

V.CONCLUSION

It is evident from Fig. 3 that the reliability of a power distribution system approaches zero as t $\rightarrow \infty$. This should be obvious, since as t gets larger, we should expect more failures and the probability of restoring all of them in t or less time obviously approaches to zero. This curve represents the percentage of system which should be available after t years of operation. Therefore, any equipment which is available for use after an extended period of time is the direct result of maintainability or the ability to repair or maintain.

Fig. (4) represents the operational availability of the power distribution system w.r.t. time and it reveals that the operational availability of the system decreases w.r.t. time and ultimately after a sufficient long interval of time. It becomes steady to the value .668965.

Fig (5) depicts that MTTF decreases w.r.t. failure rate λ' for different no. of power plants.

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